Q.3. Explain principle of least action.

OR

Write some properties of generating functions.

Q.4. Explain Lagrange Brackets.

OR

Explain canonical character of Poisson brackets.

SECTION 'C' $4 \times 10 = 40$

Long Answer questions (Word limit 400-450 words.)

Q.1. Derive Hamilton- Jacobi Equations.

OR

State and prove Local Existence Theorem.

Q.2. Using stationary phase method solve the wave equation.

OR

Explain real analytic functions and majorants.

Q.3. Explain Poincare Cartan Integral Invariant.

OR

Derive Whittaker's equations.

Q.4. Explain methods of separation of variables for finding a complete solution of the Hamilton-Jacobi equation.

OR

Explain the canonical character of a transformation.

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ROLL NO....................

MATH. 402/22

IV SEMESTER EXAMINATION, 2022

M. Sc. (MATHEMATICS)

PAPER-II

PARTIAL DIFFERENTIAL EQUATIONS & MECHANICS II

TIME: 3 HOURS **MAX.- 80**
MIN.- 16

Note: The question paper consists of three sections A, B & C. All questions are compulsory.
Section A- Attempt all multiple choice questions.
Section B- Attempt one question from each unit.
Section C- Attempt one question from each unit.

SECTION 'A' $2 \times 8 = 16$

MCQ (Multiple choice questions)

1. A complete integral of the PDE $x.Du + f(Du) = u$ is

(a) $u(x) = x + f(a) \quad (x \in U)$

(b) $u(x,a) = ax + f(a) \quad (x \in U)$

(c) $u(x,a) = x + f(ax) \quad (x \in U)$

(d) $u(x) = ax + f(ax) \quad (x \in U)$

2. The Hamiltonian corresponding to the Lagrangian $L(q,x) =$ *12mg2−ϕx is* is *Hp, x=*

(a) $\frac{1}{2m}|p| + \emptyset(x)$ (b) $\frac{1}{2m}|p|^2 + \emptyset(px)$

(c) $\frac{1}{2m}|p|^2 + \emptyset(x)$ (d) $|p^2| + \emptyset(px)$

3. If $v(y) = w(|y|)\ for\ some\ w: R \to R, r = |y|\ and\ ' = \frac{d}{dr}, then$ $\alpha v + \beta y.Dv + \Delta(v^{\gamma}) = 0$ reduce to

(a) $\alpha w + \beta r w' + (w^{r})'' + \frac{n-1}{r}(w^{r})' = 0$

(b) $\alpha w + \beta r w' + (w^{\gamma})'' + \frac{n-1}{r}(w^{r})' = 0$

(c) $w + \beta r w' + (w^{\gamma})'' + \frac{n-1}{r}(w^{\gamma})' = 0$

(d) $\alpha w + \beta r w' + (w^{\gamma})'' + \frac{1}{r}(w^{\gamma})' = 0$

4. If $u^{\epsilon}(x,t) = e^{\frac{ip^{e}(x,t)}{\epsilon}} a^{\epsilon}(x,t), (x \in R^{n}, t \geq 0)$ is a solution of the wave equation $u_{tt} - \Delta u = 0$, then p^{ϵ} is called -

(a) amplitude (b) phase

(c) geometric optics (d) all of these

5. In the action $W = \int_{t_0}^{t_1} Ldt$, the term Ldt is called -

(a) Hamilton's energy (b) Dimension of energy

(c) Motional energy (d) Elementary action

6. Which of the following represent the Poincare- Cartan integral invariant?

(a) $I = \oint \left[\sum_{i=1}^{n} pi\ \delta\ qi - L\delta t \right]$ (b) $I = \oint \left[\sum_{i=1}^{n} pi\ \delta\ qi - H\delta t \right]$

(a) $I = \oint \left[\sum_{i=1}^{n} L\ \delta qi - pi\ \delta t \right]$ (d) $I = \oint \left[\sum_{i=1}^{n} qi\ \delta\ pi - H\delta t \right]$

7. Equations $\frac{dqi}{\partial t} = \frac{\partial k}{\partial p_j}, \frac{dp_j}{dt} = -\frac{\partial k}{\partial q_j} (j = 2, - - -, n)$ are known as -

(a) Jacobi's equations (b) Lagrange's equations

(c) Whittaker's equations (d) None of these

8. If $S(t, q_i, \alpha_i)$ is some complete integral of the Hamilton – Jacobi equation $\frac{\partial s}{\partial t} + H\left(t, q_i, \frac{\partial s}{\partial q_i}\right) = 0$, then the final equations of motion of a holonomic system with the given function H may be written in the form $\frac{\partial s}{\partial q_i} = p_i, \frac{\partial s}{\partial \alpha_i} = \beta_i (i = 1, - - -, n)$ where $\alpha_i\ and\ \beta_i$ are arbitrary constants. This is statement of which of the following theorem -

(a) Hamilton's theorem (b) Jacobi's theorem

(c) Rouths theorem (d) Lee Hwa Chungs's theorem.

SECTION 'B'

$4 \times 6 = 24$

Short Answer Type Questions (Word limit 200-250 words.)

Q.1. Define Legendre Transform and prove convex quality of Hamiltonian and Lagrangian.

OR

Define Lipschitz continuity and weak solution of an IVP.

Q.2. Explain Hodograph transform.

OR

Explain Homogenization.